Sensitivity Analysis for Lexicographic Ordering in Radiation Therapy Treatment Planning

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Outline

Treatment Planning

- Dosimetrists create plans for patients, with a number of goals for the treatment outcome of varying importance
- Many models and methods have been developed to measure the quality of a plan's dose distribution
  - Challenge - tradeoffs between criteria can be difficult to quantify because some structures are more important than others
  - Tradeoffs are patient specific, making tradeoff identification difficult as well

Research Questions

- What are the tradeoffs between competing objectives in the treatment planning model?
- How can these tradeoffs be calculated efficiently and visualized or communicated in a manner valuable to physicians?

Common Technique to Treatment Planning

- Multi-criteria optimization
  - Many values are used to describe the treatment plan in the model
  - Intuitive when there are many competing objectives
  - Creates a many-dimensional Pareto frontier to realize tradeoffs

Lexicographic Ordering (LO)

- Multi-stage approach
- Uses clinical insights to prioritize treatment planning goals
  - Focuses the computational effort to clinically relevant tradeoffs
- For each stage
  - A Pareto-efficient tradeoff is plotted between competing criteria
  - The planner constrains the more important criteria accordingly, to be controlled for later stages
- Clinically, LO allows for easier interpretation of tradeoff results

[1] Jee, McShan, and Fraass (2007)

Notation

- S = set of structures
- T = set of targets
- V_s = set of voxels in structure s
- K = set of all apertures (K^* = active apertures)
- D_{kj} = dose to voxel j from aperture k at unit intensity
- u_s = upper bound on dose to voxels in structure s
- p_s = bound on EUD$_s$ after tradeoff for $s \in S$ analyzed
- α = weighting between structure EUD's, $\alpha \in [0, 1]$

Decision Variables

- z_j = dose received by voxel $j \in V$
- y_k = intensity of aperture $k \in K$
- EUD_s = Linearly-approximated EUD to structure $s \in S$

Linearly Approximating the EUD

- Equivalent Uniform Dose (EUD) can be approximated using a linear combination of the mean and max dose to the structure (mean and min dose for targets)

$$\text{EUD}_s = \gamma_s \cdot \frac{1}{|V_s|} \sum_{j \in V_s} z_j + (1 - \gamma_s) \cdot \max_{j \in V_s} z_j \qquad (s \in S \setminus T)$$

$$\text{EUD}_s = \gamma_s \cdot \frac{1}{|V_s|} \sum_{j \in V_s} z_j + (1 - \gamma_s) \cdot \min_{j \in V_s} z_j \qquad (s \in T)$$

- Motivation
  - The optimization problem can be formulated as a linear program

[3] Thieke, Bortfeld, and Küfer (2002)

General Model for LO Stage i

$$
\begin{aligned}
\min \quad & \alpha \mathrm{EUD}_{s_i} + (1-\alpha)\mathrm{EUD}_{s_{i+1}} \\
\text{s.t.} \quad & z_j = \sum_{k \in K} D_{kj} y_k && \forall j \in V \\
& z_j \leq u_s && \forall j \in V_s, s \in S \\
& \mathrm{EUD}_s = \gamma_s \cdot \frac{1}{|V_s|} \sum_{j \in V_s} z_j + (1-\gamma_s) \cdot \max_{j \in V_s} z_j && \forall s \in S \setminus T \\
& \mathrm{EUD}_s = \gamma_s \cdot \frac{1}{|V_s|} \sum_{j \in V_s} z_j + (1-\gamma_s) \cdot \min_{j \in V_s} z_j && \forall s \in T \\
& \mathrm{EUD}_{s_j} \leq p_{s_j} && s_j \in S \setminus T, \quad j \leq i-1 \\
& \mathrm{EUD}_{s_j} \geq p_{s_j} && s_j \in T, \quad j \leq i-1
\end{aligned}
$$

Generating Tradeoff Curves

- Two-phase approach
  - Phase 1 - Generate an aperture pool for K^*
  - Phase 2 - Sequentially solve LO optimization model, only allowing $y_k > 0$ when $k \in K^*$

Aperture Generation

- Generation goals
  - Generate a set of apertures large enough to produce clinically acceptable plans
  - Limit the number of apertures to keep the computational costs in Phase 2 manageable

Aperture Generation Process

- We iteratively solve the master problem with aperture set K^*, adding apertures to K^* each iteration i using Direct Aperture Optimization (DAO)
- Each iteration, the best aperture per beam is added to K^*
  - Adding only the best aperture overall produces less conformal plans
- This process continues until a desired size of K^* is reached

[2] Romeijn, Ahuja, Dempsey, and Kumar (2005)

Generating Tradeoff Curves

- For each stage i, there are two tasks:
  - Approximate tradeoff curve by solving the general stage model for various $\alpha \in [0, 1]$

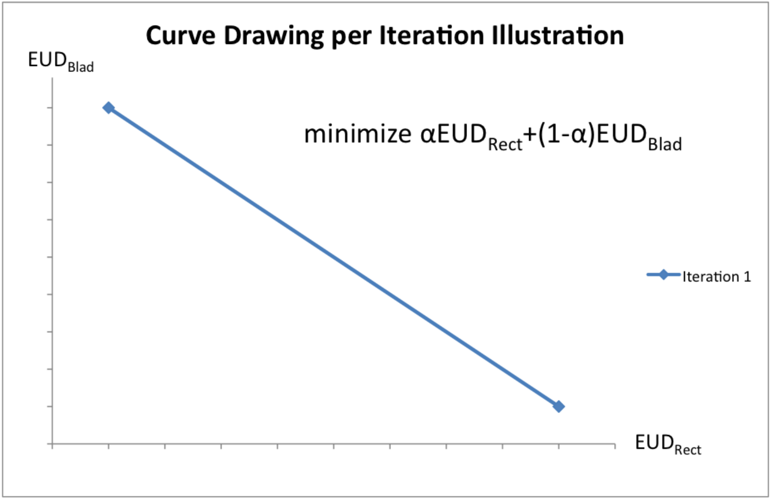
$$\min \quad \alpha \mathrm{EUD}_{s_i} + (1 - \alpha)\mathrm{EUD}_{s_{i+1}}$$

  - Select a bound for EUD_{s_i} by analyzing tradeoff curve for structure s_i
    - Add constraint $\mathrm{EUD}_{s_i} \leq p_{s_i}$ for later stages

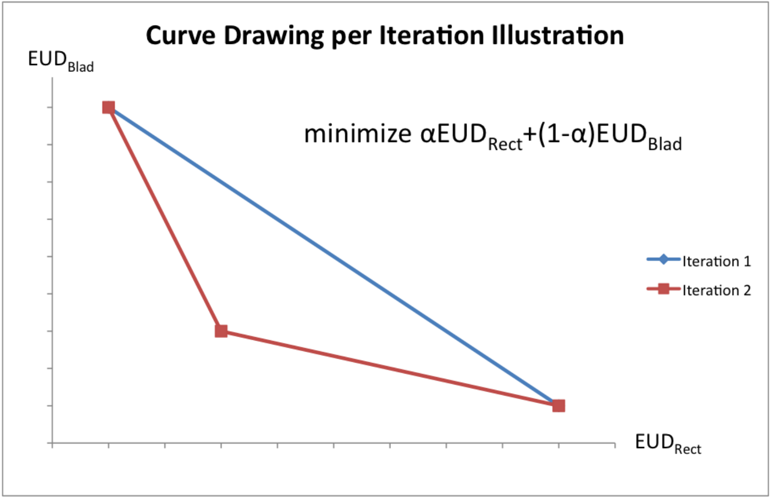
Tradeoff Curve Approximation Process

- Goal is to generate a tradeoff curve that is clinically relevant while keeping computational effort to a minimum
- We found that plotting about 8 or 9 strategically positioned solutions for different α values was sufficient

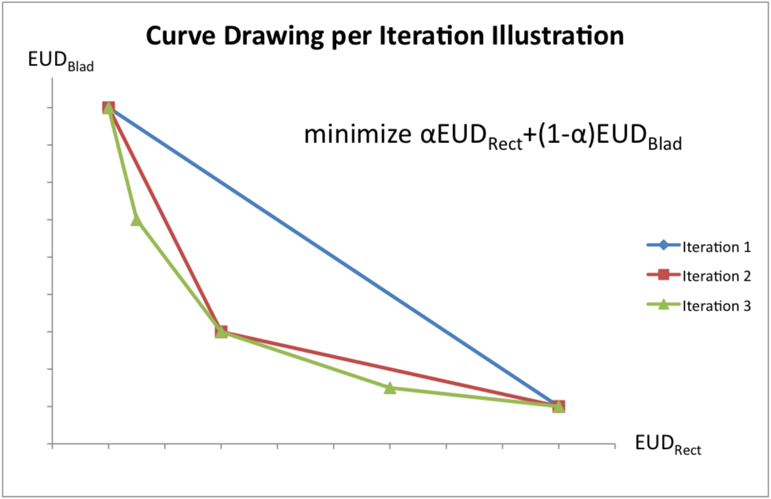
Tradeoff Curve Approximation Example



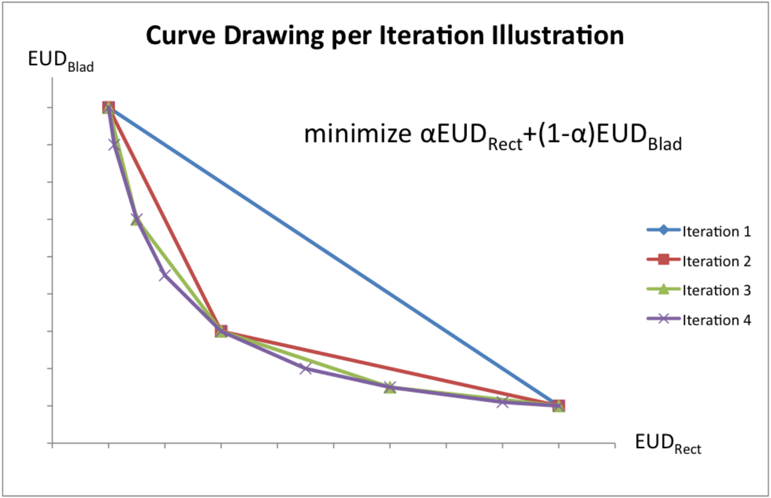
Tradeoff Curve Approximation Example



Tradeoff Curve Approximation Example



Tradeoff Curve Approximation Example



Selecting a Bound

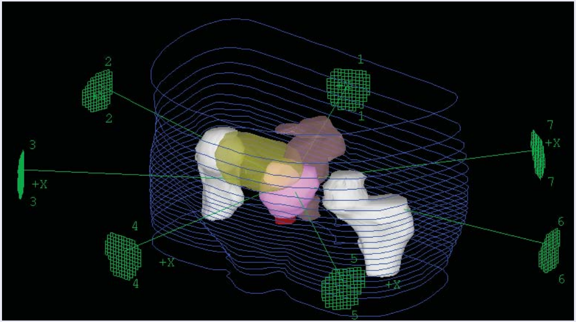
- The physician examines the tradeoff and then chooses a value, p_{s_i}, at which to constrain EUD_{s_i}
- This bound is added to the model for later stages

Application - Prostate Case

Statistics

- 7 beams
- 796 beamlets
- 44390 voxels, .5cm×.5cm

3d View



Approximate EUD Parameter γ_s

- γ_s calibrated by comparing approximate EUD to actual EUD values using a clinically acceptable dose distribution for the application case

$$\mathrm{EUD}_s = \gamma_s \cdot \frac{1}{|V_s|} \sum_{j \in V_s} z_j + (1 - \gamma_s) \cdot \max_{j \in V_s} z_j$$

Structure	PTV	Rectum	Bladder	Femora	PenileBulb
EUD Param	-5	8	7	4	1
γ_s	.3	.4	.85	.8	1

LO Model Structure

Stage	Primary Structure	Secondary Structure
1	PTV	Rectum
2	Rectum	Bladder
3	Bladder	Femora
4	Femora	Penile Bulb
5	all non-PTV voxels	-

Structure	PTV	Rectum	Bladder	Femora	Penile Bulb
EUD Goal	Max	Min	Min	Min	Min

- K^* has 84 apertures

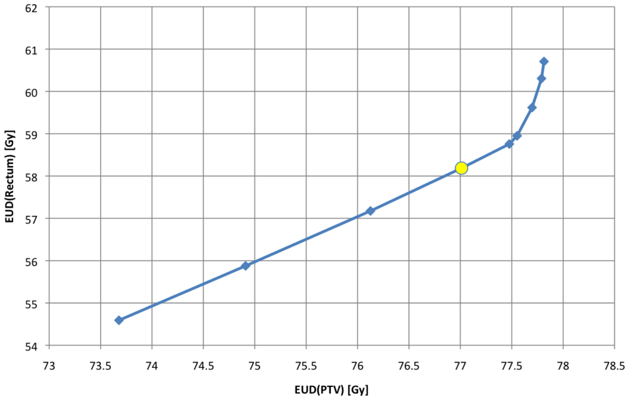
Voxel Dose Upper Bounds, u_s

Structures	PTV	Rectum	Bladder	Femora	Penile Bulb
u_s **(Gy)**	85.5	78	78	85.5	85.5

Other Structures	NT 1.5cm	NT 3cm	Other Normal
u_s **(Gy)**	83	77	65

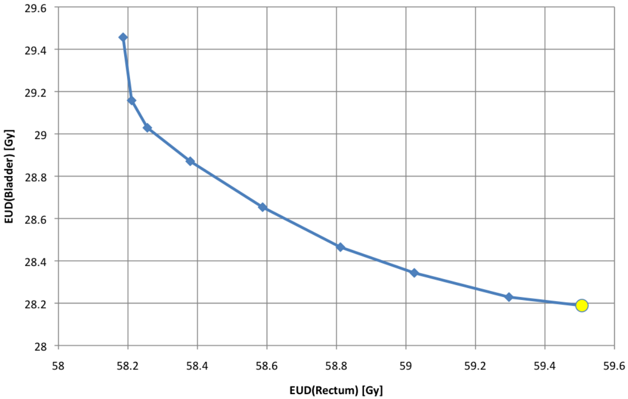
Stage 1 - PTV vs. Rectum

- Minimize $-\alpha \text{EUD}_{PTV} + (1-\alpha)\text{EUD}_{Rect}$, 133s draw time



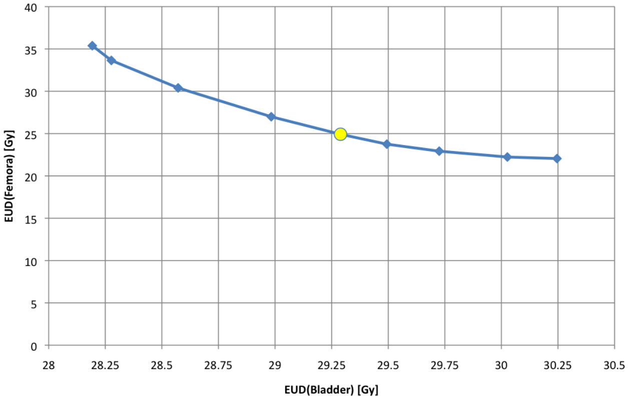
Stage 2 - Rectum vs. Bladder

- Minimize $\alpha \text{EUD}_{Rect} + (1 - \alpha)\text{EUD}_{Blad}$, 83s draw time



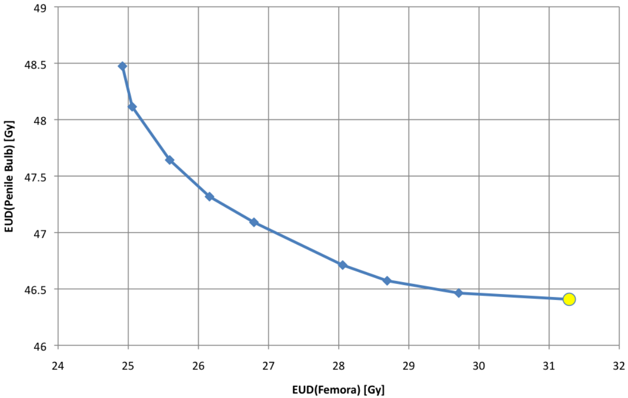
Stage 3 - Bladder vs. Femora

- Minimize $\alpha \text{EUD}_{Blad} + (1 - \alpha)\text{EUD}_{Fem}$, 183s draw time

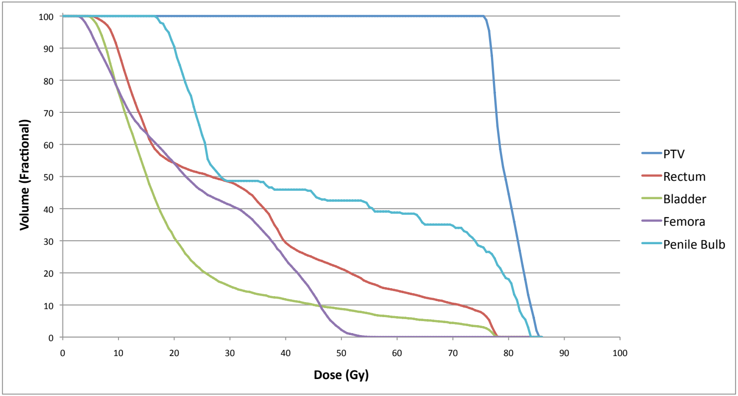


Stage 4 - Femora vs. Penile Bulb

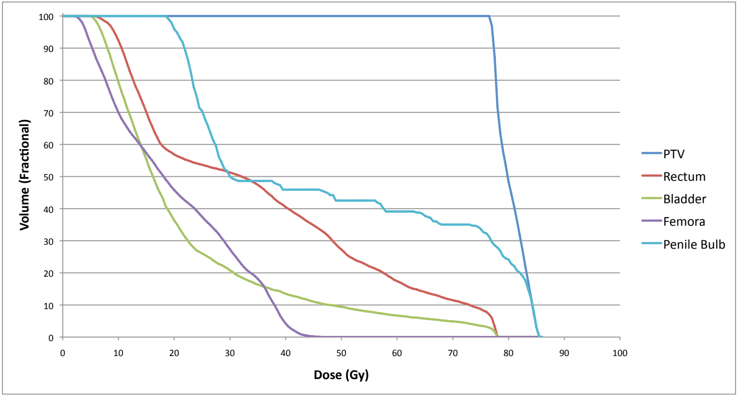
- Minimize $\alpha \text{EUD}_{Fem} + (1 - \alpha)\text{EUD}_{PB}$, 200s draw time



Dose-Volume Histogram (with chosen bounds)



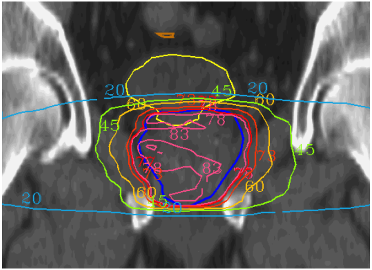
Dose-Volume Histogram (strict LO)

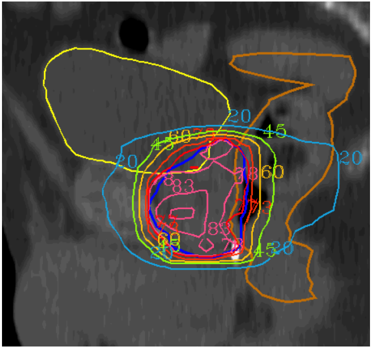


Treatment Planning Results

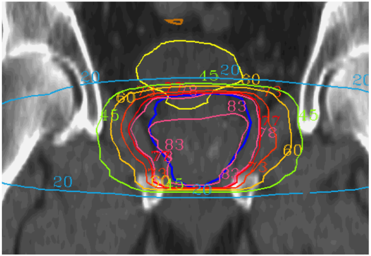
Priority	Clinical Goals (Gy)	Actual	Strict
0	$\max_{Rect} < 78$	78	78
0	$\max_{Blad} < 78$	78	78
1	$\min_{PTV} > 73.8$	75.8	76.7
2	$\text{mean}_{Rect} < 40$	31.8	34.8
2	$\text{mean}_{Blad} < 50$	20.7	22.3
3	$\min_{PTV} > 77.7$	**75.8**	**76.7**
4	$\max_{Fem} < 45$	**56.3** ($\text{mean}_{Fem} = 25.0$)	**48.6**
4	$\text{mean}_{PB} < 52.5$	46.4	48.5
4	$\max_{PB} < 77.7$	**84.0**	**85.5**

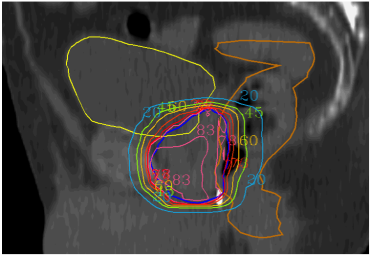
Isodose Lines (84 Apertures)





Isodose Lines (161 Apertures)





Concluding remarks

- Exploring stage-by-stage tradeoffs can help identify beneficial treatment plan alterations
- This process can be especially useful in cases with critical structures overlapping with targets
- This focuses computational effort efficiently

Future Research

- Apply technique to other regions with more impacting tradeoffs
- Study alternate means of tradeoff calculation and presentation
  - Multiple tradeoffs per stage
  - Comparing everything to PTV coverage
- Other aperture pool generation techniques
- Using GPU techniques to quicken tradeoff curve drawing process

Acknowledgements

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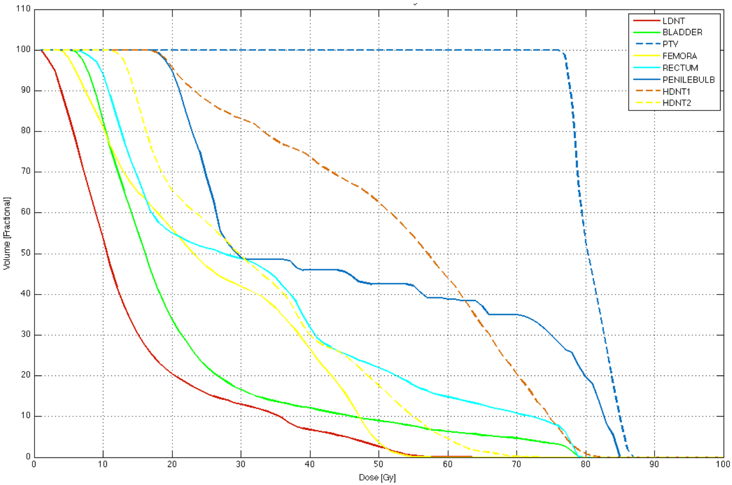
References

[1] K.-W. Jee, D.L. McShan, and B.A. Fraass. Lexicographic Ordering: Intuitive Multicriteria Optimization for IMRT. *Physics in Medicine and Biology* 52 (2007), 1845–1861.

[2] H.E. Romeijn, R.K. Ahuja, J.F. Dempsey, and A. Kumar. A column generation approach to radiation therapy treatment planning using aperature modulation. *SIAM Journal on Optimization* 15:3 (2005), 838–862.

[3] C. Thieke, T. Bortfeld, and K.-H. Küfer. Characterization of Dose Distributions Through the Max and Mean Dose Concept. *Acta Oncologica* 41 (2002), 158–161.

Dose-Volume Histogram (with chosen bounds), full



Dose-Volume Histogram (strict LO), full